Decrease in spatial coherence of light propagating in free space

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Received August 12, 1997

It is shown that a highly spatially incoherent light distribution may be generated from a highly coherent one on propagation in free space. This result essentially demonstrates that there exists an inverse of a classic result of optical coherence theory, namely, the van Cittert–Zernike theorem. The analysis also indicates that the technique of phase conjugation may be used to reverse changes in the coherence properties of light, at least those which are generated on propagation in free space.

It is well known that spatially incoherent sources can generate a fairly coherent field in large regions of space. A common example is star light entering a telescope. Star light originates in billions of atoms which radiate essentially independently by the highly random process of spontaneous emission. However, the diffraction image in the focal plane of the telescope has zero intensity minima, indicating that the light which enters the telescope is spatially highly coherent over the whole aperture of the telescope; for only under these circumstances could the intensity be completely canceled out by destructive interference in some regions of the focal plane. This result implies that spatial coherence has been generated in the process of propagation. This phenomenon has been fully clarified by optical coherence theory and is quantatively described by the so-called van Cittert–Zernike theorem (Ref. 1, Sec. 4.4.4). However, to our knowledge the possibility of appreciable *decrease* of spatial coherence on propagation in free space has not been previously demonstrated. In this Letter we show that this indeed is possible.

We begin with a known reciprocity theorem[2] for free fields.[3] Consider two free monochromatic fields, $U^{(1)}(\boldsymbol{\rho}, z)\exp(-i\omega t)$ and $U^{(2)}(\boldsymbol{\rho}, z)\exp(-i\omega t)$ $(\boldsymbol{\rho} \equiv x, y)$, of frequency ω propagating into the half-space $z > 0$ (see Fig. 1). If the field distributions in some plane $z = z_0 > 0$ are complex conjugates (denoted by an asterisk) of each other, i.e., if for all $\boldsymbol{\rho}$

$$U^{(2)}(\boldsymbol{\rho}, z_0) = [U^{(1)}(\boldsymbol{\rho}, z_0)]^*, \quad (1)$$

then

$$U^{(2)}(\boldsymbol{\rho}, z_0 + d) = [U^{(1)}(\boldsymbol{\rho}, z_0 - d)]^* \quad (2)$$

throughout the half-space, i.e., for all $d, (0 < d \leq z_0)$, and for all $\boldsymbol{\rho}$.

Consider now two statistically stationary fields represented by ensembles $\{U^{(1)}(\boldsymbol{\rho}, z)\}$ and $\{U^{(2)}(\boldsymbol{\rho}, z)\}$ of free fields, with all members of the ensembles propagating into the half-space $z > 0$. Then as a consequence of the reciprocity theorem which we just stated

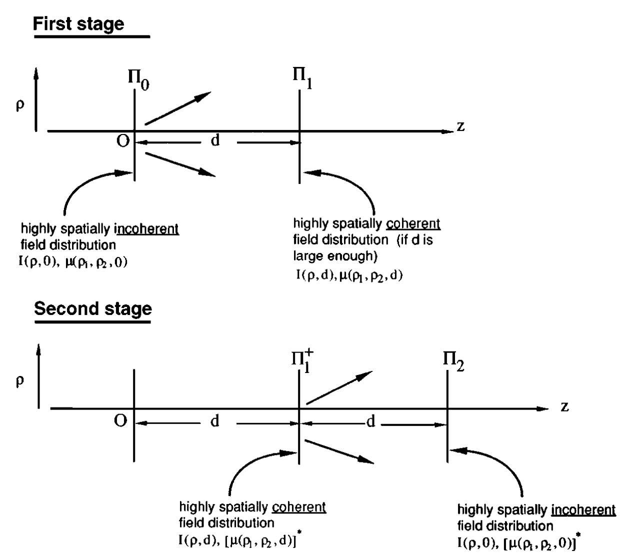

Fig. 1. Schematic illustration of generation of a spatially highly incoherent distribution from a highly coherent one on propagation in free space. The highly coherent distribution may be generated, for example, in the plane $\prod_1$, by propagating light from a highly spatially incoherent source in the plane $\prod_0$ over a large distance d in free space (First stage). The field in the plane $\prod_1$ is then phase conjugated (indicated in the figure by replacing $\prod_1$ by $\prod_1^+$) and propagates another distance d to a plane $\prod_2$. The field in the plane $\prod_2$ will be spatially highly incoherent, being essentially identical with the original field in the field $\prod_0$, except possibly for very small details (high-spatial-frequency components), which give rise to evanescent waves (whose amplitudes decay exponentially on propagation).

0146-9592/97/221672-02$10.00/0

we have the result that, if

$$\langle U^{(2)*}(\boldsymbol{\rho}_1, z_0)U^{(2)}(\boldsymbol{\rho}_2, z_0)\rangle = \langle U^{(1)*}(\boldsymbol{\rho}_1, z_0)U^{(1)}(\boldsymbol{\rho}_2, z_0)\rangle^*, \quad (3)$$

then

$$\langle U^{(2)*}(\boldsymbol{\rho}_1, z_0 + d)U^{(2)}(\boldsymbol{\rho}_2, z_0 + d)\rangle = \langle U^{(1)*}(\boldsymbol{\rho}_1, z_0 - d)U^{(1)}(\boldsymbol{\rho}_2, z_0 - d)\rangle^*, \quad (4)$$

where the angle brackets denote the ensemble average. In terms of the cross-spectral density at frequency ω [Ref. 1, Eq. (4.7-38)] we can express this result as follows: If

$$W^{(2)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z_0) = [W^{(1)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z_0)]^*, \quad (5)$$

then

$$W^{(2)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z_0 + d) = [W^{(1)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z_0 - d)]^* \quad (6)$$

for all $(0 < d \leq z_0)$ and for all $\boldsymbol{\rho}$. The formulas (5) and (6) may evidently be regarded as a generalization of the reciprocity theorem for deterministic fields, expressed by Eqs. (1) and (2), to statistically stationary fields of any state of spatial coherence.

Since the (spectral) intensity $I(\boldsymbol{\rho}, z)$ and the spectral degree of coherence $\mu(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z)$ are given by the formulas [Ref. 1, Eqs. (4.3-41) and 4.3-47)]

$$I(\boldsymbol{\rho}, z) = W(\boldsymbol{\rho}, \boldsymbol{\rho}, z), \quad (7a)$$

$$\mu(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z) = \frac{W(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z)}{\sqrt{W(\boldsymbol{\rho}_1, \boldsymbol{\rho}_1, z)}\sqrt{W(\boldsymbol{\rho}_2, \boldsymbol{\rho}_2, z)}}, \quad (7b)$$

the above theorem (with the same assumptions as before) may be expressed in the following form: If

$$I^{(2)}(\boldsymbol{\rho}, z_0) = I^{(1)}(\boldsymbol{\rho}, z_0) \quad (8)$$

and

$$\mu^{(2)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z_0) = [\mu^{(1)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z_0)]^*, \quad (9)$$

then

$$I^{(2)}(\boldsymbol{\rho}, z_0 + d) = I^{(1)}(\boldsymbol{\rho}, z_0 - d), \quad (10)$$

and

$$\mu^{(2)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z_0 + d) = [\mu^{(1)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z_0 - d)]^*. \quad (11)$$

Let us now choose $d = z_0$ in Eqs. (8)–(11). We then have the result that, if

$$I^{(2)}(\boldsymbol{\rho}, d) = I^{(1)}(\boldsymbol{\rho}, d), \quad (12)$$

and

$$\mu^{(2)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, d) = [\mu^{(1)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, d)]^*, \quad (13)$$

then

$$I^{(2)}(\boldsymbol{\rho}, 2d) = I^{(1)}(\boldsymbol{\rho}, 0), \quad (14)$$

and

$$\mu^{(2)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, 2d) = [\mu^{(1)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, 0)]^*, \quad (15)$$

Suppose now that the field in the plane $z = 0$ is highly spatially incoherent. On propagating from the plane $z = 0$ to the plane $z = d$ it will become highly spatially coherent if d is large enough, as is clear from the van Cittert–Zernike theorem. According to Eqs. (1)–(4) it follows that, if in the plane $z = d$ the field is phase conjugated [i.e., $U(\boldsymbol{\rho}, d) \to U^*(\boldsymbol{\rho}, d)$ and consequently $\mu(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, d) \to \mu^*(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, d)$] while $I(\boldsymbol{\rho}, z)$ remains unchanged, and then propagates to the plane $z = 2d$, the absolute value of the spectral degree of coherence in the plane $z = 2d$ will be the same as the absolute value of the spectral degree of coherence in the initial plane ($z = 0$), while the phases of the degrees of coherence will differ in sign. This result implies that the light distribution in the plane $z = 2d$ is spatially just as incoherent as it is was in the original plane $z = 0$, although the light is highly coherent in the plane $z = d$ from which it propagates into the half-space $z > d$. This result is illustrated schematically in Fig. 1. The phase conjugation in the plane $z = d$ can be produced by a well-known technique of nonlinear optics (see, for example, Refs. 4–6).

Tha analysis presented in this Letter demonstrates that a highly incoherent light distribution can be generated from a highly coherent one by propagation in free space. In addition, the analysis demonstrates a previously unknown feature of phase conjugation: As is well known, the phase conjugation can be used to cancel effects of distortions produced by scattering of light on deterministic media[7] and to reduce effects of wave-front aberrations and polarization distortions in laser systems.[8] These properties are often referred to as the "healing" properties of phase conjugation. Our analysis indicates, as a byproduct, that the technique of phase conjugation can also be used to cancel the changes in spatial coherence properties of light produced on propagation, at least in free space.

This research was supported by the U. S. Air Force Office of Scientific Research under grants F49620-96-1-0400 and F49620-96-1-0039 and by the U.S. Army Research Office.

References

1. L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge U. Press, Cambridge, 1995).
2. J. R. Shewell and E. Wolf, J. Opt. Soc. Am. **58,** 1596 (1968), Eqs. (4.11) and (4.12). See also R. Mittra and P. L. Ransom, in *Modern Optics*, K. Fox, ed. (Polytechnic, Brooklyn, and Wiley, New York, 1967), p. 619.
3. By a free field we mean a field which can be represented by homogeneous plane-wave modes only. General fields in a half-space include also inhomogeneous evanescent waves, whose amplitudes decay exponentially with the distance of propagation. Free fields are usually excellent approximations to actual fields, except in the immediate neighborhood of scattering bodies (cf. Ref. 1, Secs. 3.2.2 and 3.2.3).
4. D. M. Pepper, Opt. Eng. **21,** 156 (1982).
5. B. Ya Zel'dovich, N. F. Pilipetsky, and V. V. Shkunov, *Principles of Phase Conjugation* (Springer-Verlag, Berlin, 1985).
6. R. W. Boyd, *Nonlinear Optics* (Academic, New York, 1992), Sec. 6.1.
7. G. S. Agarwal, A. T. Friberg, and E. Wolf, J. Opt. Soc. Am. **73,** 529 (1983).
8. M. W. Bowers, R. W. Boyd, and A. K. Hankla, Opt. Lett. **22,** 360 (1997).